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Economics Letters 69 (2000) 201–206

**economics
letters**

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A portfolio approach to the optimal funding of pensions

Jayasri Dutta^a, Sandeep Kapur^{b,*}, J. Michael Orszag^b

^a*Faculty of Economics University of Cambridge, Cambridge, UK*

^b*Department of Economics, Birkbeck College, London W1P 2LL, UK*

Received 24 May 1999; received in revised form 21 January 2000; accepted 7 March 2000

Abstract

Fully funded pension systems are advocated for their higher rate of return, but this return is typically risky. Using a simple mean-variance model, we find that mixed funded–unfunded systems are desirable in this setting because they enable risk diversification. © 2000 Elsevier Science S.A. All rights reserved.

Keywords: Pensions; Social security; Portfolio choice

JEL classification: H55; G11

1. Introduction

The original argument for unfunded pension systems is based on the possibility of dynamic inefficiency, or capital overaccumulation (Diamond, 1965; Samuelson, 1975): individuals may save too much, and the rate of return on capital be too low relative to the growth rate of the economy. Unfunded pensions are a direct method of redistributing consumption across generations and restoring efficiency. Excessive saving may be a particular problem in situations where younger workers cannot fully insure future risks of income or mortality (Diamond, 1977).

The current case for funded pensions is made, among other things, on the grounds that the rate of return on investment exceeds income growth (e.g. Aaron, 1966; Feldstein, 1974, 1996). Therefore, in the absence of labour supply effects and changes in the tax regime, investment in funded systems yields higher returns than an unfunded system. The force of the ‘rate of return’ argument is particularly strong in countries with negative population growth and ageing populations. The cost of unfunded pensions falls asymmetrically on younger workers. Miles (1997) evaluates the practical relevance of these arguments, as well as the likely impact of transition to a funded system in the UK and comparable countries.

*Corresponding author. Tel.: +44-207-631-6405; fax: +44-207-631-6416.

E-mail address: skapur@econ.bbk.ac.uk (S. Kapur).

However, this analysis focused on the mean level of returns and ignored the variance and portfolio effects. Such effects were indeed looked at by Merton (1983) and Merton et al. (1987) as well as by Dutta et al. (1999). Earnings risks cannot be diversified directly, and partial funding allows the old to hold shares in the human capital of the young.

This paper provides a simplified treatment for some of these arguments, casting the optimal pension funding choice in a mean-variance portfolio choice framework. The optimum funding solution then is to hedge by holding a diversified set of liabilities, both funded and unfunded. Section 2 below looks at this problem. Section 3 extends the basic mean-variance model to look at what portfolio share of bonds and equities should be held in a funded system. In particular, the portfolio weights in equities and bonds might depend on the level of funding. A final section concludes.

2. A simple mean-variance model

The rate of return to a funded pension is r whereas an unfunded pension has return g where g is the growth rate in national income. We suppose a fraction w of pension liabilities are covered with funded provision and the remainder is unfunded. Then, the total pension workers will receive per unit of money contributed is: $1 + wr + (1 - w)g$. Since both r and g are random variables, the utility of mean pension income is:

$$EU(1 + wr + (1 - w)g) \quad (1)$$

Let $P = 1 + wr + (1 - w)g$ denote pension income. To obtain the optimal funded/unfunded mix, we maximize with respect to the funded share, w , obtaining:

$$\mathbf{E}(U'(P)(r - g)) = 0 \quad (2)$$

if $0 < w < 1$. In the absence of risk aversion, U' is constant, and we obtain the (Aaron, 1966) solution that it is optimal to invest in a funded system whenever $\mathbf{E}r > \mathbf{E}g$. However, with risk aversion, it may be better to hedge by using a mixture of funded and unfunded pensions. To keep the analysis simple, we focus on mean-variance utility function:

$$\mathbf{E}U(P) = \mathbf{E}P - \frac{\gamma}{2} \text{var}(P) \quad (3)$$

where γ is a risk-aversion parameter. We note that

$$\mathbf{E}P = 1 + w\mu_r + (1 - w)\mu_g \quad (4)$$

$$\text{var}(P) = w^2\sigma_r^2 + (1 - w)^2\sigma_g^2 + 2w(1 - w)\sigma_{rg} \quad (5)$$

where μ_i , σ_i^2 are the expectation and variance of any variable i and σ_{rg} the covariance of r and g . The optimal funded share is:

$$w = \frac{\mu_r - \mu_g + \gamma(\sigma_g^2 - \sigma_{rg})}{\gamma\sigma_{r-g}^2} \quad (6)$$

Table 1
Growth rates and total return on equity (1900–1989)^a

Country	GDP growth (%)		Return on equity		Covar σ_{rg}
	Mean	Variance	Mean	Variance	
	μ_g	σ_g^2	μ_r	σ_r^2	
USA	3.16	0.67	6.33	47.6	0.67
UK	1.87	0.97	4.42	60.9	4.16
France	2.42	5.37	8.4	80.7	6.57
Germany	2.88	9.72	8.93	89.4	16.2
Japan	4.42	15.71	7.76	287.9	47.52

^a Data from own calculations using 10-year averages; data on GDP from Maddison (1991); data on equity returns from Global Financial Data (1999).

How large is w for realistic parameter values? In Table 1, we report the relevant characteristics for five countries. The return on the funded component, r , is approximated by the real return on equity. As pension funds are held for relatively long periods, we report these characteristics for the period 1900–1989, using 10-year averages over this interval for our computations.

Thus the funded part of pensions (equity) has a high mean return and a high risk asset. For risk averse individuals, it is optimal to choose a mix of the two. Table 2 reports optimal values obtained from Eq. (6) for the above parameter values. As the variance–covariance parameters differ sharply across countries, so will the optimal funding levels.

As Fig. 1 illustrates, the optimal choice of w varies with the degree of risk-aversion, here parametrized by γ . It also varies with σ_{rg} , which affects the possibility of diversifying income risk in the stock market. For the US, optimal funding levels are high because the mean return on equity is high, and σ_{rg} is low. For Japan, funding levels are relatively low because σ_r^2 is large and diversification possibilities are limited.

3. Portfolio allocation in funded systems

Another consideration is that unfunded social security will affect optimal portfolio allocations of investors. As an example of this, consider a country such as Singapore or Malaysia which has a largely funded system. In such a country, there is no unfunded system to provide a risk hedge, forcing

Table 2
Risk-aversion and optimal funding

Country	Risk-aversion parameter		
	$\gamma = 0.10$	$\gamma = 0.20$	$\gamma = 0.40$
USA	77.2%	38.6%	19.2%
UK	45.1%	19.3%	6.3%
France	20.6%	9.3%	3.5%
Germany	63.5%	24.9%	5.5%
Japan	0.1%	≈ 0	≈ 0

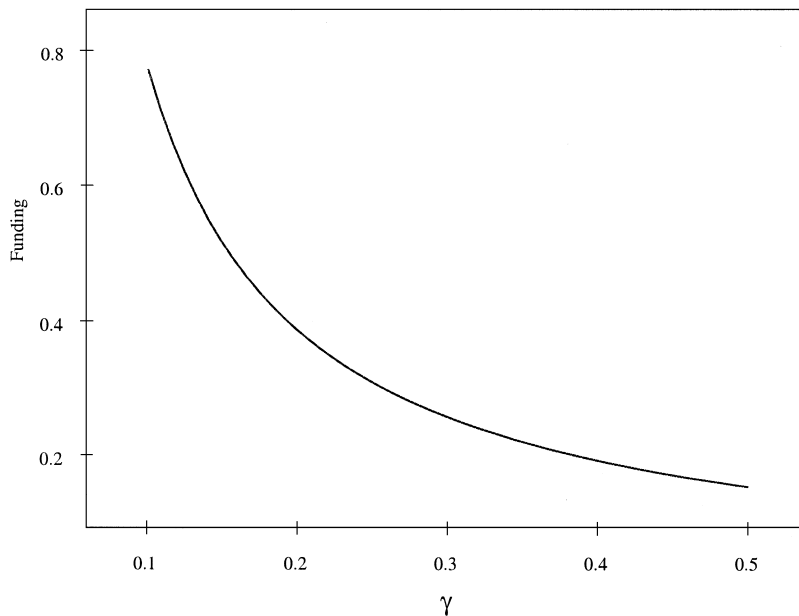


Fig. 1. The effect of changes in γ on optimal funding.

funded schemes to invest in less risky, lower return assets. While there may be other factors such as political independence of the investment board that are important, the level of risk in alternative available assets is also a crucial consideration.

In the absence of another low risk alternative, individuals and pension funds will hold government bonds which may have much lower returns than that on unfunded pensions. A common argument for funded pensions is that they will lead to more developed capital markets. One issue with these arguments is that the existence of unfunded pensions is an important risk hedge through which diversification into equities can take place.

This model is easily formalized. Each unit of pension contributions now results in a pension of:

$$P = 1 + war + w(1 - a)r_b + (1 - w)g \quad (7)$$

where a is the fraction of the funded pension allocated to equity and r_b is the return on bonds. Given fixed w , the asset allocation problem is to choose a optimally.

Using data comparable to that in Table 1, we report long-term real return on bonds.

The data in Table 3 suggest that the bonds have provided a low return in real terms, and with considerable variance. Based on these figures, the funded portion of pension systems is better allocated to equities rather than bonds. In the long run, bonds are probably dominated by a mix of unfunded and equity assets. The optimal values of a based on these figures using this data are shown in Table 4. Germany, with a relatively higher return on bonds, is one exception to the general rule that portfolios should be mainly invested in equity.

Table 3
Real return on bonds: 1900–1989^a

Country	Return on bonds	
	Mean μ_b	Var σ_b^2
USA	0.92	4.87
UK	−0.02	6.54
France	−1.9	134.7
Germany	3.3	19.4
Japan	−2.0	524

^a Calculations using 10-year averages (data from Global Financial Data, 1999).

Table 4
Portfolio allocation in funded systems: share of equity^a

Country	Funding ratio		
	$w = 0.25$	$w = 0.5$	$w = 0.75$
USA	100%	100%	88.7%
UK	100%	100%	78.2%
France	100%	100%	100%
Germany	33.7%	33.1%	33.0%
Japan	100%	100%	100%

^a Optimal a conditional on funding level; a truncated at 100%.

4. Conclusions

This paper has provided a simplified model of why funded pensions should ordinarily be combined with unfunded pensions in order to maximize the combination of risk and return. It also has provided a simple analytical framework for assessing what fraction of investments in a funded system should be funded and unfunded. This simplification comes at a cost. There is no general equilibrium wage or rate of return determination or careful modelling of issues such as early retirement and the government budget constraint. Despite the simplicity though, the message of paying more attention to risk-return tradeoffs in social security policy analysis is well-worth heeding.

References

- Aaron, H.J., 1966. The social insurance paradox. *Canadian Journal of Economics* 32, 371–374.
 Diamond, P.A., 1965. National debt in a neoclassical growth model. *American Economic Review* 55, 1026–1050.
 Diamond, P.A., 1977. A framework for social security analysis. *Journal of Public Economics* 83, 275–298.
 Dutta, J., Kapur, S., Orszag, J.M., 1999. How to fund pensions: income uncertainty and risk-aversion, Birkbeck Economics Working Paper No. 99-103.

- Feldstein, M.S., 1974. Social security, induced retirement, and aggregate capital accumulation. *Journal of Political Economy* 82, 905–926.
- Feldstein, M.S., 1996. The missing piece in policy analysis: social security reform. *American Economic Review* 86, 1–14.
- Global Financial Data, 1999; available electronically at <http://www.globalfindata.com/>
- Maddison, A., 1991. *Dynamic Forces in Capitalist Development*. Oxford University Press, Oxford.
- Merton, R.C., 1983. On the role of social security as a means for efficient risk sharing in an economy where human capital is not tradeable. In: Bodie, Z., Shoven, J. (Eds.), *Financial Aspects of the United States Pension System*. University of Chicago Press.
- Merton, R.C., Bodie, Z., Marcus, A., 1987. Pension plan integration as insurance against social security risk. In: Bodie, Z., Shoven, J., Wise, D. (Eds.), *Issues in Pension Economics*. Chicago University Press.
- Miles, D., 1997. *Financial Markets, Ageing, and Social Welfare*, Inaugural Lecture. Imperial College, University of London.
- Samuelson, P.A., 1975. Optimum social security in a life-cycle growth model. *International Economic Review* 16, 538–544.